

LU & QR Flop Count

Because why not



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LU Factorization by Gaussian Elimination

For , take row operation: , which is flops each.

Then for , take row operation: , which is flops each.

Continue until saturated.

(After which, triangular matrices can be computed by back/forward substitution, which the order is .)

QR Factorization by Gram-Schmidt

Take where each represents a column of .

By Gram-Schmidt:

This can be inverted to write

Take , where

To count flops, note that takes flops to compute ( from and 1 from square root).

takes flops. ( for , 1 from dividing it by . Note that is already calculated from calculating .)

takes flops.

QR Factorization by Householder Reflections

Seen in lecture note.

Note that computing inner product and outer product for , flop counts are and respectively.

For computing , flop count is , where each term comes from calculating , division from , calculating outer product , calculating scalar-with-matrix product , and subtracting off from the identity matrix ().

To choose such that has only the top entry, it takes flops ( for the inner product , 1 for square root, and for .

Suppose it is calculated that , then we now need to calculate . Note that the product of successive householder matrix here is of the form:

(Where bold fonted are vectors.)

Note that no flop is needed to compute the first column. takes flops and takes